- อินเวอร์ส ของ เมทริกซ์ ระบบ
เพื่อหา ไปใช้ในการแก้สมการ เชิง เส้น

$$3x_1 + 2x_2 + 5x_3 = 10 \quad - (1)$$

$$-x_1 + 4x_2 - 2x_3 = 5 \quad - (2)$$

$$-2x_1 + 3x_2 + 10x_3 = -2 \quad - (3)$$

จัดให้อยู่รูป ให้ อยู่ ในระบบสมการ ของ เมทริกซ์

$$Ax = b \Rightarrow x = A^{-1}b$$

$$\begin{bmatrix} 3 & 2 & 5 \\ -1 & 4 & -2 \\ -2 & 3 & 10 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 10 \\ 5 \\ -2 \end{Bmatrix}$$

$$A^{-1} = \frac{adj(A)}{det(A)}$$

$$adj(A) = C_A^T$$

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}_{3\times 3}$$

$$B = \begin{bmatrix} B_{11} \\ B_{21} \\ B_{31} \end{bmatrix}_{3\times 1}$$

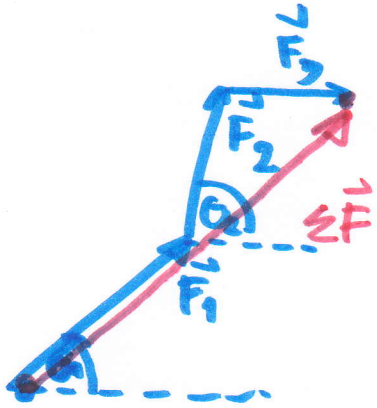
$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix},$$

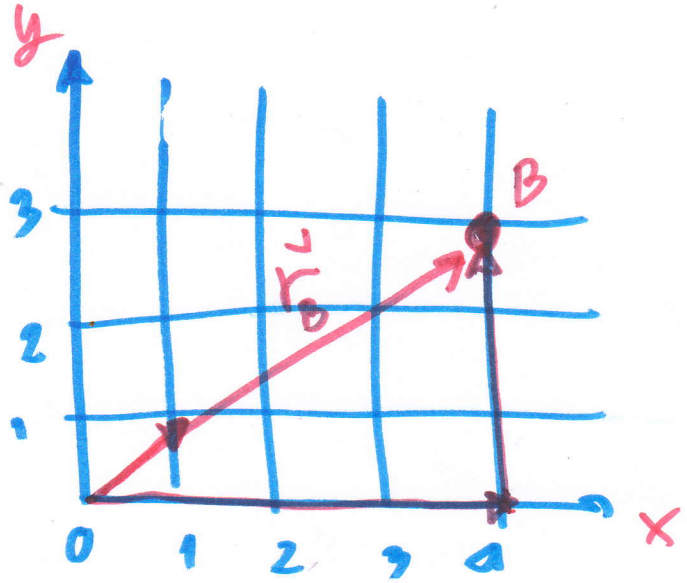
$$A \times B = \begin{bmatrix} 1\times(-1) + (2\times 0) + (1\times 2) \\ 2\times(-1) + (3\times 0) + (1\times 2) \\ 2\times(-1) + (1\times 0) + (3\times 2) \end{bmatrix}$$

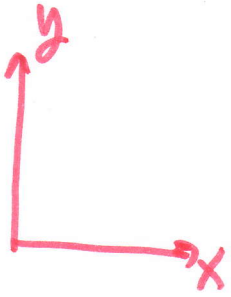
$$= \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$$

• Static equilibrium

$\Sigma \vec{F}$



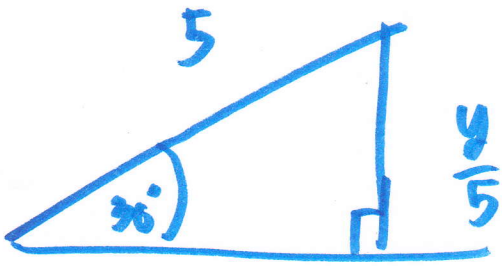




$$\vec{r}_m = 4\vec{i} + 3\vec{j}$$

$$|\vec{r}_m|\,\vec{u}_{r_m} = \vec{r}_m$$

$$\vec{u}_{r_m} = \frac{\vec{r}_m}{|\vec{r}_m|} = \frac{1}{\sqrt{4^2+3^2}}\left[4\vec{i}+3\vec{j}\right]$$

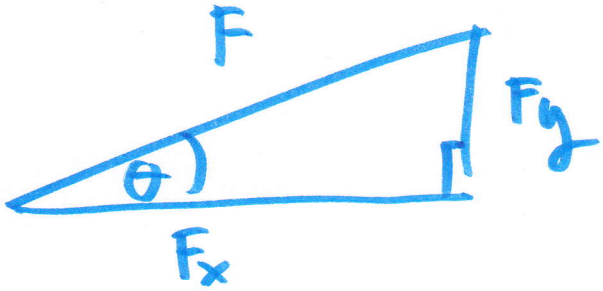


$\frac{y}{5} = \sin 30$

$y = 5\sin 30$

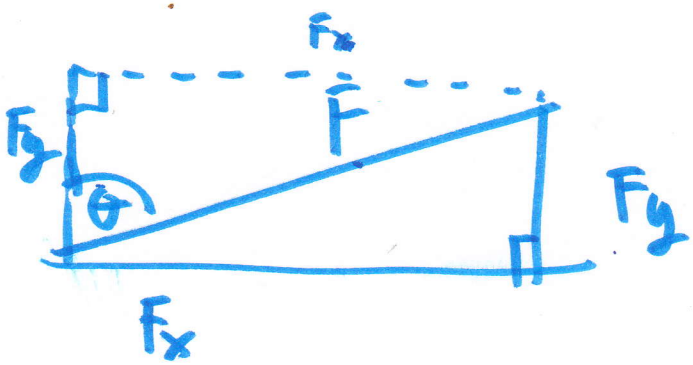
$\frac{x}{5} = \cos 30$

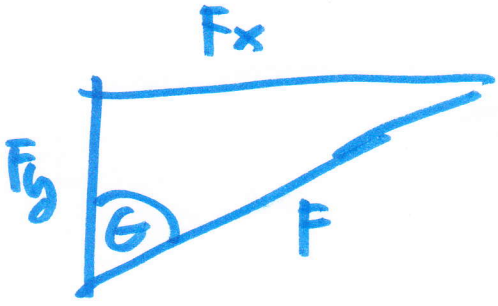
$x = 5\cos 30$



$\frac{F_x}{F} = \cos\theta$ | $\frac{F_y}{F} = \sin\theta$

$F_x = F\cos\theta$ | $F_y = F\sin\theta$

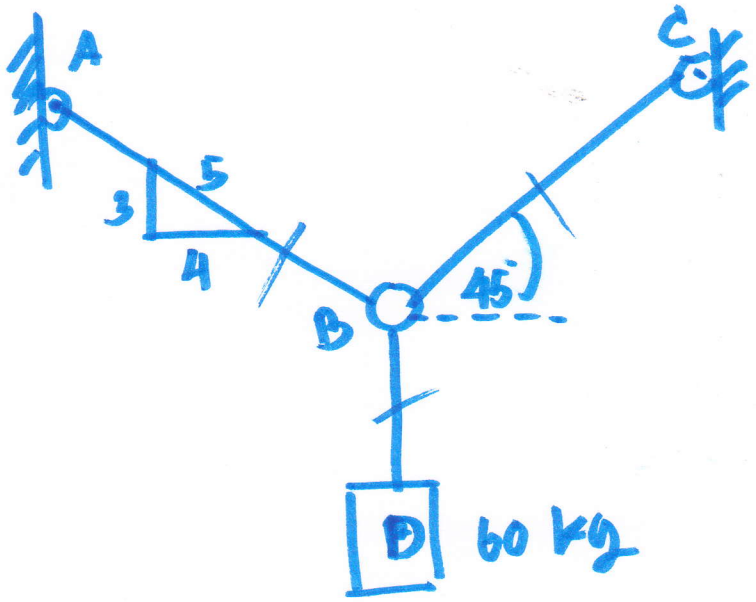




$\frac{F_x}{F} = \sin\theta$ | $\frac{F_y}{F} = \cos\theta$

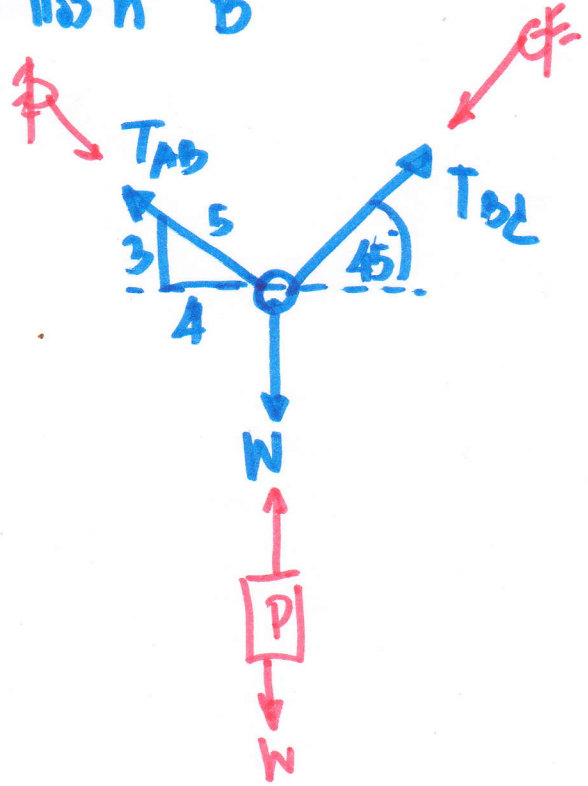
$F_x = F\sin\theta$ | $F_y = F\cos\theta$

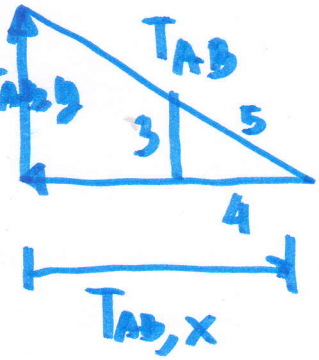
- Determine the tension in cables AB and BC necessary to support the 60-kg cylinder



จัดอนุภาค ที่ B

FBD





$\sum \vec{F} = 0$

$\sum F_x = 0$

$\sum F_y = 0$

ตัวแปร $\Rightarrow T_{AB}, T_{BC}$

$\xrightarrow{+} \sum F_x = 0$

$-\frac{4}{5} T_{AB} + T_{BC} \cos 45^\circ = 0$

$+\uparrow \Sigma F_y = 0$

$$\frac{3}{5} T_{AB} + T_{BC} \sin 45^\circ - 60 \times 9.81 = 0$$

$$\frac{3}{5} T_{AB} + \sin 45^\circ T_{BC} = 588.6 \quad -(2)$$

$$-\frac{4}{5} T_{AB} + \cos 45^\circ T_{BC} = 0 \quad -(1)$$

ข้อสรุป สมการ เชิงเส้น ที่เขียนอยู่ ในรูป ของ ระบบสมการ เมทริกซ์

$$Ax = b$$

$$\begin{bmatrix} -\frac{4}{5} & \cos 45^\circ \\ \frac{3}{5} & \sin 45^\circ \end{bmatrix} \begin{Bmatrix} T_{AB} \\ T_{BC} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 588.6 \end{Bmatrix}$$

A x b

$$A^{-1} = \frac{adj(A)}{det(A)} \checkmark$$

$$det(A) = \begin{vmatrix} -\frac{4}{5} & \cos 45^\circ \\ \frac{3}{5} & \sin 45^\circ \end{vmatrix} = \left(-\frac{4}{5} \times \sin 45^\circ\right) - \left(\frac{3}{5} \times \cos 45^\circ\right)$$

$$= -0.99$$

$$det(A) = -0.99$$

$$adj(A) = C_A^T$$

$$C_A = \begin{bmatrix} M_{11} & \ominus M_{12} \\ \ominus M_{21} & M_{22} \end{bmatrix}$$

$$= \begin{bmatrix} \sin 45^\circ & -\frac{3}{5} \\ -\cos 45^\circ & -\frac{4}{5} \end{bmatrix}$$

$$C_A^T = \begin{bmatrix} \sin 45^\circ & -\cos 45^\circ \\ -\frac{3}{5} & -\frac{4}{5} \end{bmatrix}$$

$$A^{-1} = \frac{adj(A)}{det(A)} = \frac{C_A^T}{det(A)}$$

$$= -\frac{1}{0.99} \begin{bmatrix} \sin 45^\circ & -\cos 45^\circ \\ -\frac{3}{5} & -\frac{4}{5} \end{bmatrix}$$

$$X = \begin{Bmatrix} T_{AB} \\ T_{BC} \end{Bmatrix} = -\frac{1}{0.99} \begin{bmatrix} \sin 45^\circ & -\cos 45^\circ \\ -\frac{3}{5} & -\frac{4}{5} \end{bmatrix}_{2\times 2} \begin{Bmatrix} 0 \\ 588.6 \end{Bmatrix}_{2\times 1}$$

$$\begin{Bmatrix} T_{AB} \\ T_{BC} \end{Bmatrix} = -\frac{1}{0.99} \begin{Bmatrix} (\sin 45^\circ \times 0) + (-\cos 45^\circ \times 588.6) \\ (-\frac{3}{5} \times 0) + (-\frac{4}{5} \times 588.6) \end{Bmatrix}$$

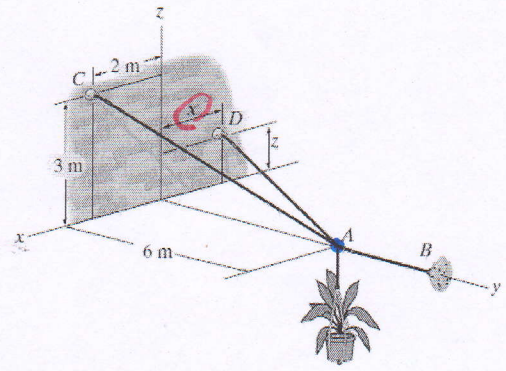
$$\begin{Bmatrix} T_{AB} \\ T_{BC} \end{Bmatrix} = -\frac{1}{0.99} \begin{Bmatrix} -416.2 \\ -470.88 \end{Bmatrix}$$

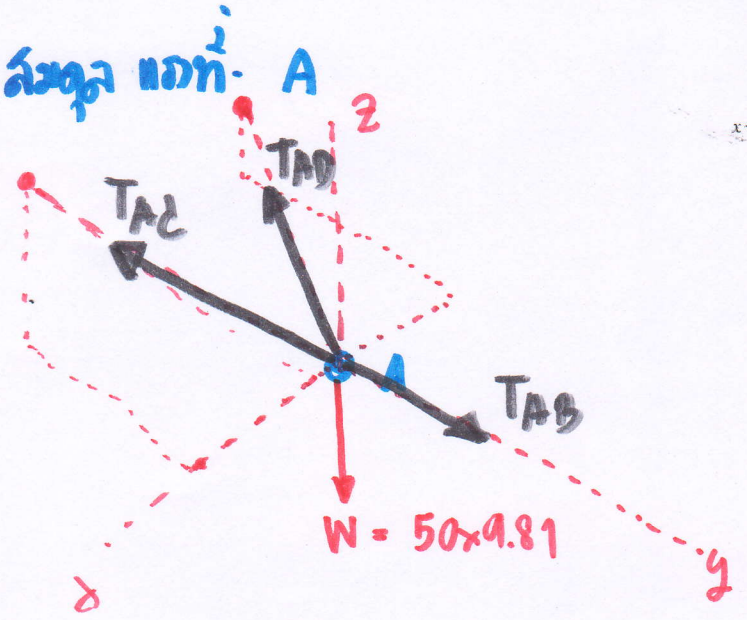
$$\begin{Bmatrix} T_{AB} \\ T_{BC} \end{Bmatrix} = \begin{Bmatrix} 420.4 \\ 475.64 \end{Bmatrix}$$

$$T_{AB} = 420.4 \text{ N} \quad \text{※}$$

$$T_{BC} = 475.64 \text{ N} \quad \text{※}$$

3–54. If the mass of the flowerpot is 50 kg, determine the tension developed in each wire for equilibrium. Set $x = 1.5$ m and $z = 2$ m.





$$\vec{r}_{AD} = -2\vec{i} - 6\vec{j} + 2\vec{k}$$

-1.5

$$\vec{r}_{AC} = 2\vec{i} - 6\vec{j} + 3\vec{k}$$

$$\vec{u}_{r_{AD}} = \frac{1}{|\vec{r}_{AD}|}\vec{r}_{AD} = -0.23\vec{i} - 0.92\vec{j} + 0.31\vec{k}$$

$$\vec{u}_{r_{AC}} = \frac{1}{|\vec{r}_{AC}|}\vec{r}_{AC} = 0.28\vec{i} - 0.86\vec{j} + 0.43\vec{k}$$

$$\vec{T}_{AD} = T_{AD}\vec{u}_{r_{AD}} = -0.23T_{AD}\vec{i} - 0.92T_{AD}\vec{j} + 0.31T_{AD}\vec{k}$$

$$\vec{T}_{AC} = T_{AC}\vec{u}_{r_{AC}} = 0.28\,T_{AC}\vec{i} - 0.86\,T_{AC}\vec{j} + 0.43\,T_{AC}\vec{k}$$

$$\vec{T}_{AD} = T_{AD}\vec{j}$$

$$\vec{W} = -50\times 9.81\vec{k}$$

$$\vec{W} = -490.5\vec{k}$$

$$\Sigma\vec{F} = 0$$

$$\vec{T}_{AD} + \vec{T}_{AC} + \vec{T}_{AB} + \vec{W} = 0$$

$$\Sigma F_x = 0\ ,\ \Sigma F_y = 0\ ,\ \Sigma F_z = 0$$

✓ $0.28\,T_{AC} - 0.23\,T_{AD} = 0$ — ①

✓ $-0.86\,T_{AC} - 0.92\,T_{AD} + T_{AB} = 0$ — ②

$0.43\,T_{AC} + 0.31\,T_{AD} - 490.5 = 0$ — ③

✓ $0.43\,T_{AC} + 0.31\,T_{AD} = 490.5$ — ③

$$\begin{bmatrix} 0.28 & -0.23 & 0 \\ -0.86 & -0.92 & 1 \\ 0.43 & 0.31 & 0 \end{bmatrix} \begin{Bmatrix} T_{AC} \\ T_{AD} \\ T_{AB} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 490.5 \end{Bmatrix}$$

$$C_A = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

$$= \begin{bmatrix} (-1)^{1+1} M_{11} & (-1)^{1+2} M_{12} & (-1)^{1+3} M_{13} \\ (-1)^{2+1} M_{21} & (-1)^{2+2} M_{22} & (-1)^{2+3} M_{23} \\ (-1)^{3+1} M_{31} & (-1)^{3+2} M_{32} & (-1)^{3+3} M_{33} \end{bmatrix}$$

$$C_A = \begin{bmatrix} M_{11} & -M_{12} & M_{13} \\ -M_{21} & M_{22} & -M_{23} \\ M_{31} & -M_{32} & M_{33} \end{bmatrix}$$

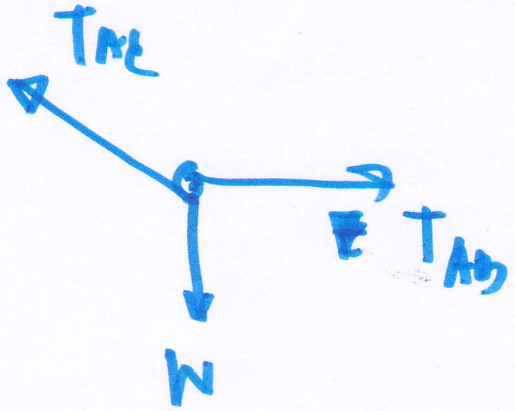
$$C_A^T = \begin{bmatrix} M_{11} & -M_{21} & M_{31} \\ -M_{12} & M_{22} & -M_{32} \\ M_{13} & -M_{23} & M_{33} \end{bmatrix}$$

$$A^{-1} = \frac{adj(A)}{det(A)} = \frac{C_A^T}{det(A)}$$

$$X = A^{-1} b$$

Quiz

$T_{AB} = kx$



ต้องหา T_{AC} , T_{AB}

$$T_{AB} - T_{AC}\cos 30^\circ = 0 \quad \text{— ①}$$

$$T_{AC}\sin 30^\circ = 78.48 \quad \text{— ②}$$

$$\begin{bmatrix} 1 & -\cos 30^\circ \\ 0 & \sin 30^\circ \end{bmatrix} \begin{Bmatrix} T_{AB} \\ T_{AC} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 78.48 \end{Bmatrix}$$

จาก T_{AB} , T_{AC}

$$T_{AB} = 135.93$$

$$T_{AC} = 156.96$$